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Multi-synchronization of chaos via linear output feedback strategy

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Abstract

Multi-synchronization of chaotic systems based on the master-slave scheme as an extension of the dual synchronization problem is introduced. It is assumed that the only information available from the master systems is a linear combination of their state vectors. The design procedure for multi-synchronization through output feedback strategy is described and the sufficient condition is given. The performance of the proposed algorithm is numerically examined by applying it to the Chen–Lorenz–Rossler and the Duffing–Van der Pol chaotic systems. Simulation results show the effectiveness of the proposed scheme.

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1. Introduction

Since Pecora and Carrols have presented their synchronization schemes [1,2], synchronization of chaotic systems has become one of the most important topics in nonlinear dynamics and chaos [3]. Chaos synchronization has found applications in many fields of science and technology such as biology, chemistry, secret communication, cryptography and nonlinear oscillations. Synchronization of two identical chaotic systems with different initial conditions was realized in electronic circuits for the first time [1,2]. However, recently chaos synchronization of two different chaotic systems is widely investigated [4,5]. In all of the mentioned works, synchronization is considered for systems consisting of one master and one slave system. Recently dual synchronization has been used for communication [6–8]. From the nonlinear dynamic point of view, synchronization of chaos in multiple pairs of one-way coupled oscillators is a very interesting topic, and is related to identification of chaos from mixed chaotic waves [6]. In dual synchronization problem, there are two slave systems that must be synchronized with two master systems. The state vectors of the master systems are combined linearly to generate a signal that is fed to the slave systems. Multiplexing chaotic signal and synchronizing more than one pair of chaotic systems using only one communication channel is investigated in [7]. In [8] dual synchronization between two one dimensional master and slave chaotic maps by a scalar error signal is introduced. Dual synchronization in Colpitts electronic oscillators has been tested experimentally in [6]. Dual synchronization of two different chaotic systems is studied in [9]. In that work the output signal of the master

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system is a scalar signal constructed by a linear combination of their states. In [9] for designing a synchronization scheme, it is assumed that all states of master and slave systems are available and they are used for updating the gain of synchronizing signal. Dual and dual cross synchronization of chaotic external cavity laser diodes have been investigated in [10]. In [6,11] dual synchronization of chaos in two pairs of one-way coupled microchip lasers using only one transmission channel have been studied. In [12] dual synchronization using the time dependent linear output feedback is presented.

In this paper problem of multi-synchronization via output feedback strategy is investigated. The present work is indeed an extension of our previous work [12]. The output signal of the master systems, i.e. the input signal to the slave systems, is a multiplexed signal generated by linear combination of master systems states. It is assumed that this signal is the only information available from the master systems and is used for complete state synchronization of master and slave systems. The error between master and slave output signals is multiplied by a variable feedback gain and used for multi-synchronization. The feedback gain is designed such that synchronization between all state variables of slave and master systems is achieved. The sufficient condition for multi-synchronization using the proposed control strategy is given. Finally, the effectiveness of the proposed scheme is demonstrated through computer simulation.

2. Problem statement

Consider the following N chaotic systems as the master dynamics:

The dynamic of i th master system is given by:

$$\dot{X}^{(i)} = f^{(i)}(t, X^{(i)}), \quad i = 1, \dots, N \quad (1)$$

where $X^{(i)} = [X_1^{(i)} \dots X_n^{(i)}]^T$ are the state vectors of the master systems and $f^{(i)} \in C^1(\mathcal{R}^+ \times \mathcal{R}^n, \mathcal{R}^n)$ are known functions. By linear combination of master systems state vectors, a chaotic signal is generated as given below:

$$v_m = \sum_{i=1}^N A^{(i)} X^{(i)} = C\eta \quad (2)$$

where $A^{(i)}, i = 1, \dots, N$ is an $n \times n$ matrix with known constant elements; $C = [A^{(1)} \ A^{(2)} \ \dots \ A^{(N)}]$ and $\eta = [(X^{(1)})^T \ (X^{(2)})^T \ \dots \ (X^{(N)})^T]^T$. The above generated signal is fed to the slave systems which have the same dynamics as the master systems:

The dynamic of i th slave system is given by:

$$\dot{x}^{(i)} = f^{(i)}(t, x^{(i)}) + u^{(i)}, \quad i = 1, 2, \dots, N \quad (3)$$

where $x^{(i)} = [x_1^{(i)} \dots x_n^{(i)}]^T$, and $u^{(i)}$ is the manipulated variable, i.e. the control action. The main goal is to synchronize each of the slave system with its corresponding master system, i.e.

$$\lim_{t \rightarrow \infty} \|x^{(i)}(t) - X^{(i)}(t)\| = 0 \quad (4)$$

where $\|\dots\|$ denotes the Euclidian norm. It should be noted that the only signal available from the master system is the vector v_m defined by Eq. (2); and using this signal, the multi-synchronization objective must be achieved. The schematic diagram of the multi-synchronization scheme is shown in Fig. 1.

3. Multi-synchronization by output feedback strategy

By defining $\Phi = [(f^1)^T \ (f^2)^T \ \dots \ (f^N)^T]^T$, one can rewrite Eq. (1) in the following form:

$$\dot{\eta} = \Phi(t, \eta). \quad (5)$$

Similarly by setting $\xi = [(x^{(1)})^T \ (x^{(2)})^T \ \dots \ (x^{(N)})^T]^T$, Eq. (3) can be rewritten as:

$$\dot{\xi} = \Phi(t, \xi) + U \quad (6)$$

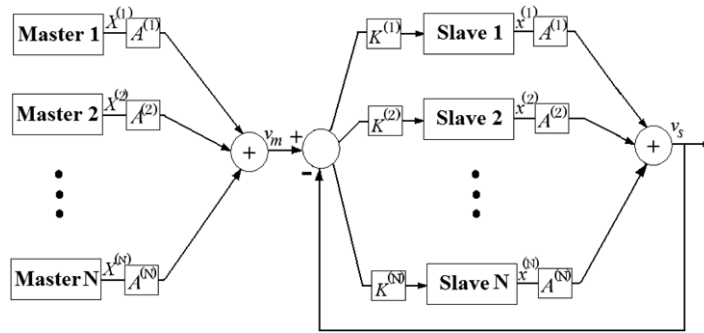


Fig. 1. Multi-synchronization scheme.

where $U^T = [(u^{(1)})^T, (u^{(2)})^T, \dots, (u^{(N)})^T]^T$. The feedback signal from the slave system, v_s , is generated as given below:

$$v_s = \sum_{i=1}^N A^{(i)} x^{(i)} = C\xi. \quad (7)$$

Considering the block diagram shown in Fig. 1, the error signal for multi-synchronization is given by:

$$\begin{aligned} e &= v_s - v_m \\ &= C(\xi - \eta). \end{aligned} \quad (8)$$

The proportional output feedback is proposed for chaos synchronization, and the closed loop equation is given below:

$$\dot{x}^{(i)} = f^{(i)}(t, x^{(i)}) + K^{(i)}e, \quad i = 1, 2, \dots, N \Rightarrow \dot{\xi} = \Phi(t, \xi) + Ke \quad (9)$$

where $K = [(K^{(1)})^T \ (K^{(2)})^T \ \dots \ (K^{(N)})^T]^T$ is the proportional feedback gain which is designed to synchronize all chaotic systems simultaneously. Setting $E = \xi - \eta$, the error dynamics is obtained as:

$$\begin{aligned} \dot{\xi} - \dot{\eta} = \dot{E} &= \Phi(t, \xi) - \Phi(t, \eta) + Ke \\ &= \Phi(t, \xi) - \Phi(t, \xi - E) + KC(\xi - \eta) \\ &= \Phi(t, \xi) - \Phi(t, \xi - E) + KCE. \end{aligned} \quad (10)$$

The dual synchronization goal given by Eq. (4) is equivalent to:

$$\lim_{t \rightarrow \infty} \|\xi - \eta\| = 0 \Leftrightarrow \lim_{t \rightarrow \infty} \|E\| = 0. \quad (11)$$

It means that the proportional gain K must be designed in such a way that the origin becomes an asymptotic stable equilibrium point of dynamics (10), i.e. $\lim_{t \rightarrow \infty} E = 0$. $\Phi(., .)$ is a C^1 function, and its first order Taylor expansion yields:

$$\begin{aligned} \Phi(t, \xi) - \Phi(t, \xi - E) &= \frac{\partial \Phi(t, \xi)}{\partial \xi} E + h.o.t \\ &= \tilde{G}(t, \xi(t))E + h.o.t \\ &\triangleq G(t)E + h.o.t \end{aligned} \quad (12)$$

where $h.o.t$ denotes the higher order terms of the series or the residue of the series. Substituting Eq. (12) into Eq. (10) yields:

$$\dot{E} = [G(t) + KC]E + h.o.t. \quad (13)$$

If one can design a control gain K such that the linear part of the above dynamics is asymptotically stable, then it is locally asymptotically stable. This condition will be satisfied if pair of $(G(t), C)$ satisfies the observability condition at any time t according to following theorem [13].

Theorem 1. *The linear time varying system given by $\dot{E} = G(t)E$, $e = CE$, is completely observable if for any t , there exists $\tau > t$ such that the observability matrix defined as:*

$$H(\tau, t) \triangleq \int_t^\tau \Psi^T(s, t) C^T C \Psi(s, t) ds \quad (14)$$

where $\Psi(., .)$ is the state transition matrix, is nonsingular; i.e. $\det(H(\tau, t)) \neq 0$. ♣

Applying Theorem 1 to the linearized error equation $\dot{E} = G(t)E$, $e = CE$, implies that if the pair $(G(t), C)$ satisfies the observability condition, then there exists a matrix K which guarantees the local stability of Eq. (13) ($\lim_{t \rightarrow \infty} E(t) = 0$).

As it is shown by Eq. (12), $G(t)$ depends on the states of the slave systems, hence is a time varying matrix. Different methods can be used for designing K [14]; one of them is satisfying the following equation:

$$\forall t_0 > 0, \quad \lim_{T \rightarrow \infty} \int_{t_0}^{t_0+T} \mu[G(t) + KC] dt = -\infty \quad (15)$$

where $\mu(.)$ is a matrix measure [14]. If the above condition is satisfied then $E = 0$ is locally asymptotically stable.

In Eq. (15) one may use another matrix measure which is defined for an arbitrary matrix M as [14]:

$$\mu_2(M) = \lambda_{\max} \left[\frac{1}{2} (M + M^*) \right] \quad (16)$$

where M^* is the transpose-conjugate of M . Setting $M = G(t) + KC^T$ Eq. (16) yields:

$$\mu_2[G(t) + KC^T] = \lambda_{\max} \left[\frac{1}{2} (G(t) + G^T(t) + KC + C^T K^T) \right]. \quad (17)$$

Remark 1. To obtain a proper gain K to satisfy Eq. (15), one can select K such that:

$$\lambda_{\max} \left[\frac{1}{2} (G(t) + G^T(t) + KC + C^T K^T) \right] = m \quad (18)$$

where m is a negative real number. Eq. (18) implies that condition (15) is satisfied, and also provides a controller design criterion. ♣

It must be noted that $G(t)$ is a state dependent matrix, and consequently the feedback gain K obtained from Eq. (18) is a time varying matrix, and due to continuity of $G(t)$ with respect to $\xi(t)$ and t , K is also continuous with respect to $\xi(t)$ and t . In addition, the feedback gain K will be a function of time and the states of the slave systems, hence for calculating the synchronizing control signal, Ke , individual states of the master system are not required.

4. Simulation and results

Example 1. Multi-synchronization of the Chen–Lorenz–Rossler dynamical systems:

In this example the multi-synchronization of the Lorenz, Rossler and Chen dynamical systems by using the proposed method, is investigated. The dynamic equations of these systems are given by:

Master 1: Chen system:

$$\begin{aligned} \dot{X}_1^{(1)} &= a_1(X_2^{(1)} - X_1^{(1)}) \\ \dot{X}_2^{(1)} &= -a_1X_1^{(1)} + b_1(X_1^{(1)} + X_2^{(1)}) - X_1^{(1)}X_3^{(1)} \\ \dot{X}_3^{(1)} &= X_1^{(1)}X_2^{(1)} - c_1X_3^{(1)}. \end{aligned} \quad (19)$$

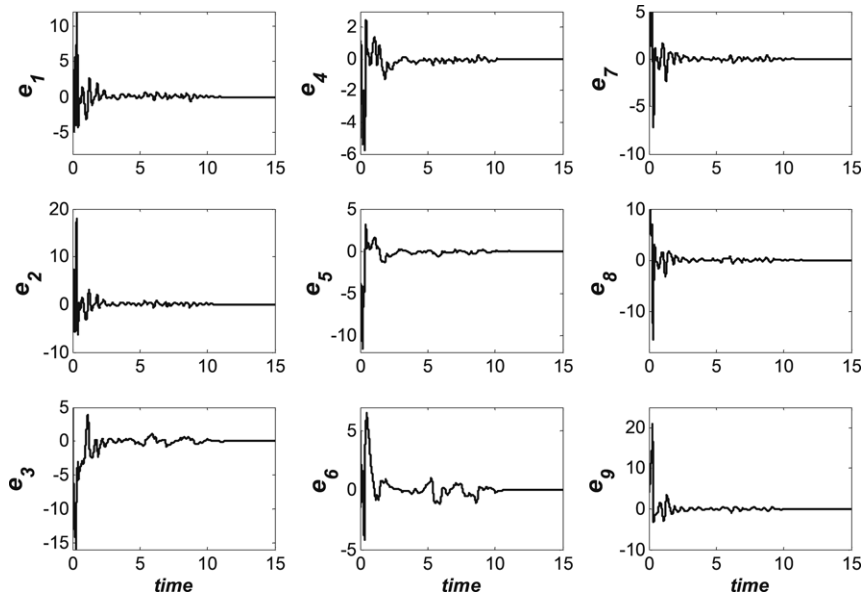


Fig. 2. Multi-synchronization results for the Chen–Lorenz–Rossler systems with v_m given by (23).

Master 2: Lorenz system:

$$\begin{aligned}\dot{X}_1^{(2)} &= a_2(X_2^{(2)} - X_1^{(2)}) \\ \dot{X}_2^{(2)} &= c_2 X_1^{(2)} - X_2^{(2)} - X_1^{(2)} X_3^{(2)} \\ \dot{X}_3^{(2)} &= X_1^{(2)} X_2^{(2)} - b_2 X_3^{(2)}.\end{aligned}\quad (20)$$

Master 3: Rossler system:

$$\begin{aligned}\dot{X}_1^{(3)} &= -(X_2^{(3)} + X_3^{(3)}) \\ \dot{X}_2^{(3)} &= X_1^{(3)} + a_3 X_2^{(3)} \\ \dot{X}_3^{(3)} &= X_1^{(3)} X_3^{(3)} - c_3 X_3^{(3)} + b_3.\end{aligned}\quad (21)$$

The slave systems are the same as the master systems, and similar to Eq. (3) their states are denoted by $x^{(i)}$, $i = 1, 2, 3$. The $G(t)$ matrix of Eq. (12) is obtained as:

$$G(t) = \begin{bmatrix} -a_1 & a_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -a_1 + b_1 - x_3^{(1)}(t) & b_1 & -x_1^{(1)}(t) & 0 & 0 & 0 & 0 & 0 & 0 \\ x_2^{(1)}(t) & x_1^{(1)}(t) & -c_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -a_2 & a_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c - x_3^{(2)}(t) & -1 & -x_1^{(2)}(t) & 0 & 0 & 0 \\ 0 & 0 & 0 & x_2^{(2)}(t) & x_1^{(2)}(t) & -b_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & a_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x_3^{(3)}(t) & 0 & x_1^{(3)}(t) - c_3 \end{bmatrix}. \quad (22)$$

The system parameters are set to $a_1 = 35$, $b_1 = 28$, $c_1 = 3$, $a_2 = 10$, $b_2 = 8/3$, $c_2 = 28$, $a_3 = 0.2$, $b_3 = 0.2$ and $c_3 = 5$ which results to chaotic behavior of all three systems.

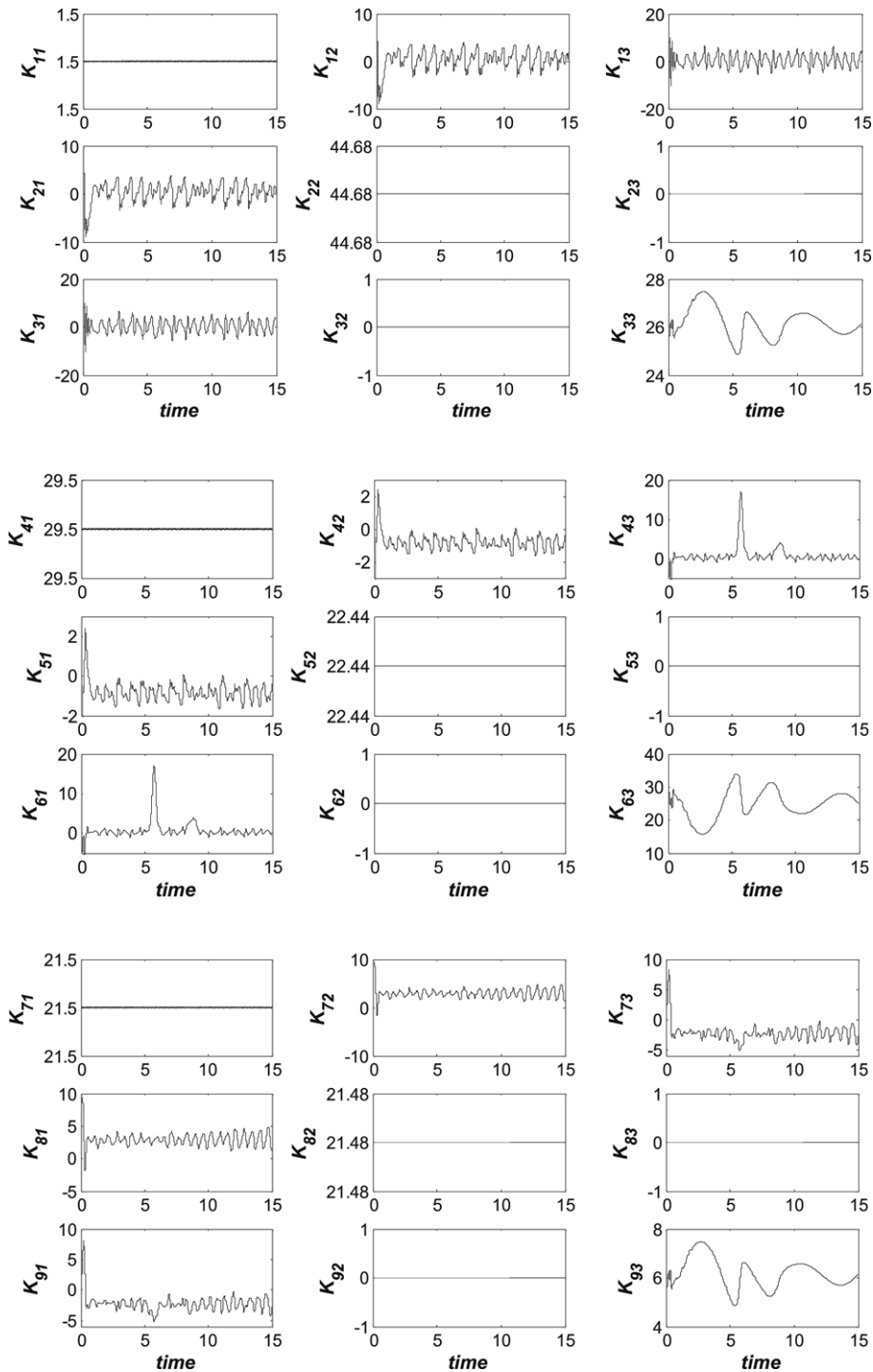


Fig. 3. Feedback gains vs. time for multi-synchronization of Chen–Lorenz–Rossler systems with v_m given by (23).

For the first case, the output signal of the master systems is assumed to be:

$$v_m = \begin{bmatrix} X_1^{(1)} + X_1^{(2)} + X_1^{(3)} \\ X_2^{(1)} + X_2^{(2)} + X_2^{(3)} \\ X_3^{(1)} + X_3^{(2)} + X_3^{(3)} \end{bmatrix}. \quad (23)$$

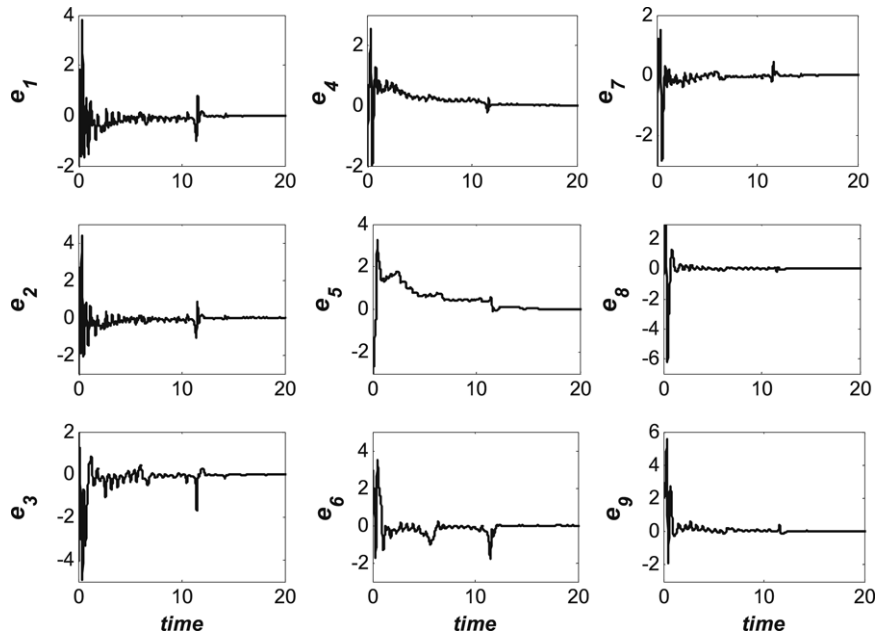


Fig. 4. Multi-synchronization results for the Chen–Lorenz–Rossler systems with v_m given by (25).

So the matrices $A^{(i)}$, $i = 1, 2, 3$ and C are:

$$A^{(i)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad i = 1, 2, 3 \Rightarrow C = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}. \quad (24)$$

The initial conditions are set to $X_1^{(1)}(0) = 5$, $X_2^{(1)}(0) = 5$, $X_3^{(1)}(0) = -5$, $X_1^{(2)}(0) = 5$, $X_2^{(2)}(0) = 5$, $X_3^{(2)}(0) = -5$, $X_1^{(3)}(0) = 5$, $X_2^{(3)}(0) = -5$, $X_3^{(3)}(0) = -5$ and $Y_j^{(i)}(0) = 0$, $i, j = 1, 2, 3$. m in Eq. (18) is set to -2 . Simulation results are shown in Fig. 2. e_i 's are the elements of the E vector. The elements of the feedback gain resulting from Eq. (18) are plotted in Fig. 3.

For the second case, the output signal of the master systems is assumed to be:

$$v_m = \begin{bmatrix} X_1^{(1)} - 2X_3^{(1)} + X_1^{(2)} + X_1^{(3)} - 3X_3^{(3)} \\ X_2^{(1)} - 2X_1^{(2)} + X_2^{(2)} + X_2^{(3)} \\ -3X_1^{(1)} + X_3^{(1)} + X_3^{(2)} + 2X_1^{(3)} + X_3^{(3)} \end{bmatrix}. \quad (25)$$

So the matrices $A^{(i)}$, $i = 1, 2, 3$ and C are:

$$A^{(1)} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}, \quad A^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad A^{(3)} = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \quad (26)$$

$$\Rightarrow C = \begin{bmatrix} 1 & 0 & -2 & 1 & 0 & 0 & 1 & 0 & -3 \\ 0 & 1 & 0 & -2 & 1 & 0 & 0 & 1 & 0 \\ -3 & 0 & 1 & 0 & 0 & 1 & 2 & 0 & 1 \end{bmatrix}.$$

The initial conditions are set to $X_1^{(1)}(0) = 3$, $X_2^{(1)}(0) = 3$, $X_3^{(1)}(0) = -3$, $X_1^{(2)}(0) = 3$, $X_2^{(2)}(0) = 3$, $X_3^{(2)}(0) = -3$, $X_1^{(3)}(0) = 3$, $X_2^{(3)}(0) = -3$, $X_3^{(3)}(0) = -3$ and $Y_j^{(i)}(0) = 0$, $i, j = 1, 2, 3$. m in Eq. (18) is set to -2 . Simulation results and the feedback gain $K(t)$ are shown in Figs. 4 and 5

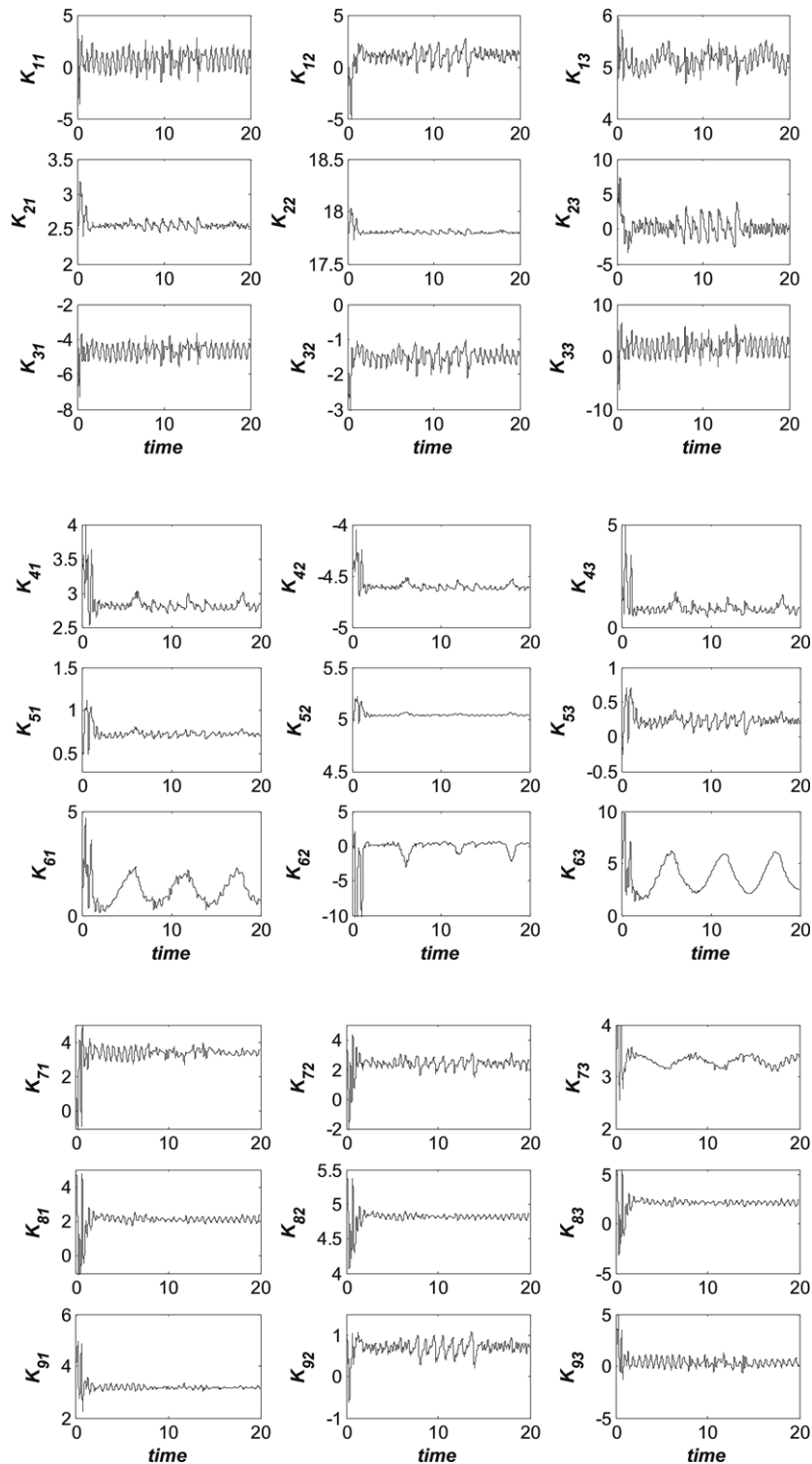


Fig. 5. Feedback gains vs. time for multi-synchronization of Chen–Lorenz–Rossler systems with v_m given by (25).

As can be seen from Figs. 2 and 4 all error signals have converged to zero.

Example 2. Dual synchronization of the Duffing–Van der Pol dynamical systems:

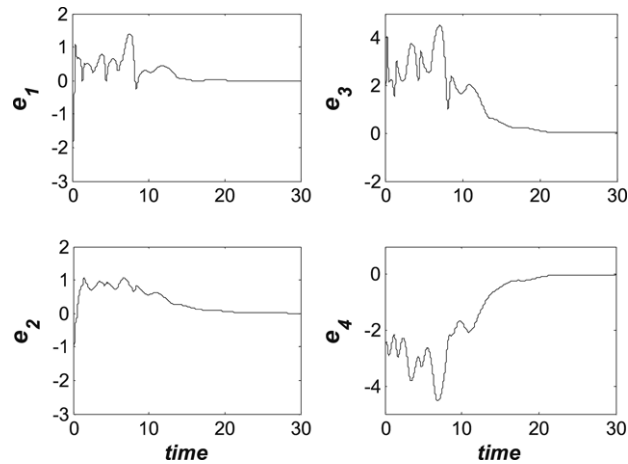


Fig. 6. Dual synchronization results for Duffing–Van der Pol systems with v_m given by (30).

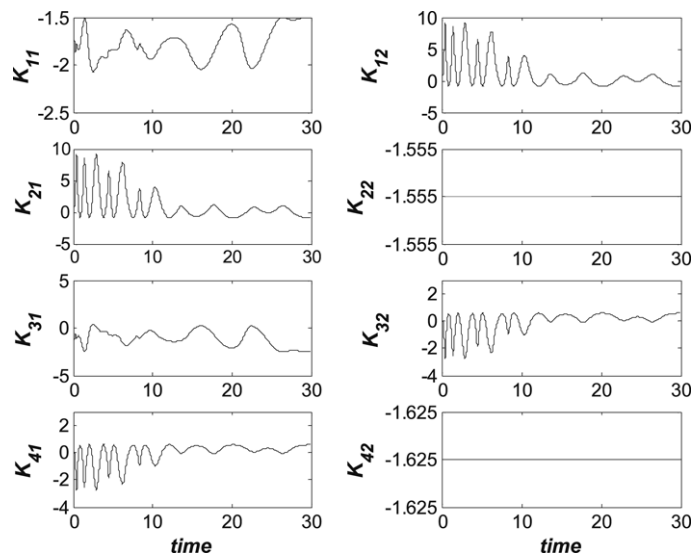


Fig. 7. Feedback gains vs. time for dual-synchronization of Duffing–Van der Pol systems with v_m given by (30).

In the second example, the master and the slave systems are chosen as follows:

Master 1: Duffing system

$$\begin{aligned}\dot{X}_1^{(1)} &= X_2^{(1)} \\ \dot{X}_2^{(1)} &= aX_1^{(1)} + b(X_1^{(1)})^3 + cX_2^{(1)} + f_0 \cos t.\end{aligned}\quad (27)$$

Master 2: Van der Pol system

$$\begin{aligned}\dot{X}_1^{(2)} &= X_1^{(2)} - (X_1^{(2)})^3/3 - X_2^{(2)} + f_1 \cos t \\ \dot{X}_2^{(2)} &= \gamma(X_1^{(2)} + \alpha - \beta X_2^{(2)}).\end{aligned}\quad (28)$$

Using the first order Taylor approximation, the $G(t)$ matrix is obtained as:

$$G(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a + 3b(x_1^{(1)}(t))^2 & c & 0 & 0 \\ 0 & 0 & 1 - (x_1^{(2)}(t))^2 & -1 \\ 0 & 0 & \gamma & -\gamma\beta \end{bmatrix}.\quad (29)$$

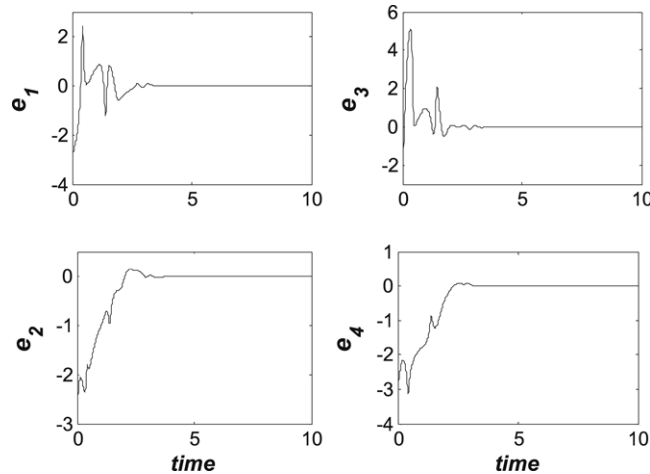


Fig. 8. Dual synchronization results for Duffing–Van der Pol systems with v_m given by (31).

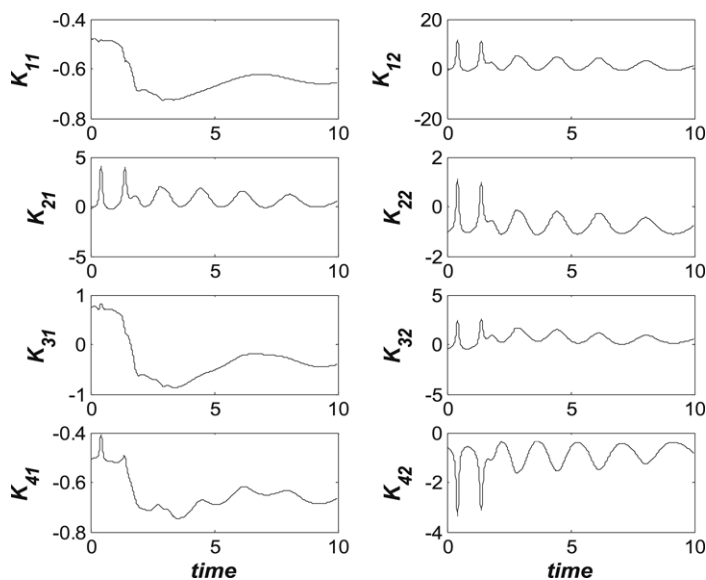


Fig. 9. Feedback gains vs. time for dual-synchronization of Duffing–Van der Pol systems with v_m given by (31).

By setting $a = 1$, $b = -1$, $c = -0.15$, $f_0 = 0.3$, $\alpha = 0.7$, $\beta = 0.8$, $\gamma = 0.1$ and $f_1 = 0.74$ both systems show chaotic behaviors.

Two cases for simulation are considered. For the first case the output signal of the master systems is selected to be:

$$v_m = \begin{bmatrix} X_1^{(1)} + X_1^{(2)} \\ X_2^{(1)} + X_2^{(2)} \end{bmatrix} \quad (30)$$

and the initial conditions are set to $X_1^{(1)}(0) = 3$, $X_2^{(1)}(0) = 3$, $X_1^{(2)}(0) = 3$ and $X_2^{(2)}(0) = 3$, and $Y_j^{(i)}(0) = 0$, $i, j = 1, 2$. For dual synchronization, the proposed method is applied by setting $m = -2$ in Eq. (18). The results are shown in Fig. 6. The feedback gains obtained from Eq. (18) are shown in Fig. 7.

For the second case v_m is set to:

$$v_m = \begin{bmatrix} 2X_1^{(1)} - X_2^{(1)} - 3X_1^{(2)} + 2X_2^{(2)} \\ 2X_2^{(1)} + X_1^{(2)} + X_2^{(2)} \end{bmatrix} \quad (31)$$

with the same initial conditions. The results of synchronization and variations of feedback gains are shown in Figs. 8 and 9.

As can be seen from Figs. 6 and 8 the slave systems have been synchronized with the master systems.

5. Conclusion

In this paper problem of multi-synchronization of chaotic systems by time varying proportional output feedback is investigated. The output signal of the master systems is a linear combination of their state vectors, and the output signal of the slave systems is generated in the same way. Using theory of linear time varying systems, a sufficient condition for multi-synchronization is given. Based on the criterion, obtained from the observability condition, a control algorithm is introduced. The proposed scheme is applied for synchronization of the Chen–Lorenz–Rossler and the Duffing–Van der Pol systems as two case studies to show the high performance of the proposed method.

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